Modeling and Analysis of
An Electromagnetic Launch System

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by
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To my Parents, wife and kids.
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Abstract

As there is a renewed interest in the hypervelocity impact and electromagnetic launch (EML), Indian Institute of Science is developing an Electromagnetic Launch System (ELS) to accelerate low mass projectile to a high velocity, for peaceful space applications. Theoretical estimates are needed for a better understanding of the system and for optimization of different system parameters. As a first step towards this, numerical simulation is done using a COMSOL\textsuperscript{TM} 3D model of the geometry to assess the design adequacy and to predict the body forces developed in the armature of the ELS. The quasi-static simulation in which the armature is assumed as stationary, gives the distribution of current density over the assembly and the forces developed on the rails and armature. The force on the armature is checked for the design adequacy as per the muzzle velocity and projectile mass specifications. In addition, an analytical study on the system dynamics is performed assuming Euler Bernoulli beam theory for the rails using Lagrangian formulation, incorporating the frictional force due to the moving armature. Characteristics of flexural and axial wave propagation is also studied and spectral finite element formulation is done to derive the dynamic stiffness matrix as a function of frequency to analyze the stability of the system for different frequencies.
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Chapter 1

Introduction

Efforts to use electromagnetism to launch objects at high velocities were started long back in 1844. Since then many advances occurred in this field, especially with the renewed interest in recent years. A railgun is the simplest device to launch projectiles of comparatively low mass with a very high velocity, using the principle of electromagnetism. The layout of an Electromagnetic Launch System (ELS) consists of two parallel conductive rails which are shorted using an armature, which is free to move between the rails. When current passes through the ELS assembly, magnetic field will be setup and the armature and rails experience electromagnetic force. The rails are contained using mechanical clamps and the force on armature will eject it out of the assembly at a high velocity, depending on the magnitude of current [1].

1.1 Theory of Electromagnetic Launch System (ELS)

An ELS consists of a pair of conducting rails separated by a distance and with the rails connected to the positive and negative sides of a power source supplying a high current. A conducting armature shorts the gap between the rails, completing the electrical circuit. The high current through the rails sets up a magnetic field and its interaction with the electric field produce forces on the conductors, i.e. the rails and the armature (fig.1.1). The rails will be contained using mechanical clamps and hence movement will be
restrained. But the armature is always free to move along the rails with a sliding contact and hence will be thrown out of the rails as the force developed exceeds the frictional force between the armature and rails. The direction of magnetic field established and forces developed will be as per the right hand rule. The current in both the directions will produce a force in the armature which will propel it out of the rails. The force produced in the rails will be in such a way that it will repel the rails. See fig.1.2 for details.

The forces developed in the rails can be estimated from the equation

\[ dF_E = J \times B, \]

where \( dF_E \) is the force per unit volume, \( J \) is the current density, \( B \) is the magnetic field, \( \times \) represents the vector cross product.

The force in Eq.(1.1) can also be expressed as

\[ F_E = 1/2L'I^2, \]

(1.2)
where $F_E$ is the force, $L'$ is the Inductance per unit length along the pair of rails and $I$ is the current.

It is clear from Eqs.(1.1) and (1.2) that the forces produced on the rails and armature depends on the current density and the magnetic field established. Also a typical value of $L'$ is around $0.4 \times 10^{-6}$ H/m for a simple rail structure. Hence in order to impart hypervelocity to the armature the supplied current should be large enough. So a very large current of short duration is preferred for ELSs. This in turn produces very high forces on the rails, which will try to push it apart. As the rails are mechanically clamped together, the associated stress developed in the rails will be of very high intensity for hypervelocity ELSs.

Another way to increase the force and muzzle velocity of projectile is to enhance the magnetic field established. The magnetic field developed across the armature can be enhanced in various ways. A core with high magnetic permeability may be used to increase the magnetic field density, although this may not be helpful if the ELS operates well past the saturation point of the material. A strong permanent magnet circuit which
can produce a field higher than the ELS operating field, may be used to provide a magnetic field different from that provided by the ELS. Additional rails that do not make contact with the armature may be placed near the main rails in order to increase the magnetic field density. Any of these options can also be implemented with a completely separate synchronized circuit.

The ELS has so many losses associated with its functioning. So it is important to reduce the associated losses to improve the system efficiency. The improvement of ELS system efficiency has been seriously studied by many researchers and there has been a sincere effort to understand the loss associated with the ELS. McNab [2] and Johnson [3] details the loss mechanisms, methods to reduce input power and improvement of efficiency.

1.2 Types of Electromagnetic Launch Systems

The ELS discussed in previous sections is the simplest form, in which two conducting parallel rails is shorted by an armature. There are limitations for this model. The muzzle velocity that can be attained by this kind of simpler systems are much smaller than the requirement for Electromagnetic Launch (EML) to space or other applications. Hence there are enhancements over the simple one, made in order to increase the muzzle velocity and efficiency of the system.

1.2.1 Distributed Power Source

As it is difficult to build a single power supply of large capacity, ELSs with distributed power source are used in many experiments. This will help in enhancing the efficiency of the system. But in this case, the complexity associated with the switching circuit design will be more complex. The circuit has to be properly designed to switch the supply from one unit to the other as the armature passes the sections powered by each supply [4, 5].
1.2.2 ELS with Plasma Armature

Here a non conductive material will be used for the projectile and a thin metal foil is placed on the back of the projectile. High current through this foil vaporizes it and forms a plasma layer behind the projectile. This plasma layer carries the current and completes the circuit. Plasma armature gun has the advantage that the mechanical wear and tear and associated loses are less for such systems.

The Institute for Advanced Technology (IAT), Texas has undertaken research in the area of hypervelocity and plasma-driven EML. One such effort of plasma driven launch experiment and hardware details are presented in ref. [6].

1.2.3 Coil Guns

It is another variant of ELS in which one or more coils are used to establish the magnetic field to accelerate a magnetic projectile to very high velocity and this totally avoids the rail armature sliding contact [1]. Here the armature current is induced by the same principles as in electric transformer with no physical contact between conducting elements. In its simplest form, pulsed induction launcher consists of a single stationary drive coil and an electrically independent, movable coaxial armature coil. When the drive current is pulsed through the drive coil, the magnetic field generated by the drive current ($I_1$) induces a current ($I_2$) in the armature coil, generating a magnetic field in the opposite direction of the field in the drive coil. The armature coil is accelerated by the mutual repulsion given by $F_E = I_1I_2dM/dx$, where $M$ is the mutual inductance and $x$ is the separation between the coils. A large number of drive coils energized simultaneously as the armature pass through it can increase the efficiency of the system.

1.3 Applications of ELS

Potential applications envisaged are many for an ELS, however the research and experiments have to go a long way to prove the feasibility and practical use of EML in all those areas. Listed below are few of them in the lime light
1.3.1 Military Applications

ELS is of particular interest in defense applications. These systems can replace large artillery and will have the advantages of being light, easy to transport and easy to handle. Another positive factor is that because of the hypervelocity, the missiles launched using ELS will not be that easy to intercept and the directional stability will also be better. It is being projected as an important part of Strategic Defense Initiative (SDI) [7]. SDI is a US government program responsible for development of a space based system to defend attacks by ballistic missiles. ELS can be used to intercept such missiles and it can even be used to assure protection from rogue asteroids heading towards Earth.

1.3.2 Non-military Applications

The system can be used to launch objects and micro satellites to space in a more effective manner, once the associated technologies are fully developed. The concept of launching material to space has been the subject of keynote presentations and several symposium publications since the inception of the EML Symposia. The primary attraction of electromagnetic launch to space is the projected savings in the cost per kilogram of launching material to low earth orbit. Projected savings by factors of from 300 to 1000 times appears possible, but it is clearly one of the most difficult applications of the technology. Ian R McNab et al has discussed several such issues in refs. [6], [8], [9]. The initial U.S. interest in electromagnetic launch was stimulated by the early designs by Henry Kolm and Gerard O’Neill of the mass driver to launch material from the surface of the moon to a space station in earth orbit [10]. The researchers in the area sincerely believe that, in future, low cost access and commercialization of space will be enabled by this launch technology.

EML can also be used to initiate fusion reactions. Fusion occurs when two small atomic nuclei combine together to form a larger nucleus, a process that releases large amounts of energy. Atomic nuclei must be traveling at enormous velocities for this to happen. It is proposes to use railguns to fire pellets of fusible material at each other. The impact of the high-velocity pellets would create immense temperatures and pressures,
enabling fusion to occur. Onozuka M. et al [11] have developed a railgun pellet injection system for fusion experimental devices. Using a low electric energy railgun system, hydrogen pellet acceleration tests have been conducted to investigate the application of the electromagnetic railgun system for high speed pellet injection into fusion plasmas.

1.4 Associated Engineering Challenges

The ELS is promising for use in different applications as discussed in previous section. But there are many associated technologies, which need to be understood to a greater depth. This calls for more detailed research and extensive experimental studies in this field. Few major challenges identified are listed below.

1.4.1 Power Supply and Switching

The power supply to fire ELS must provide a very large current of short duration. It must also operate at a high enough voltage to drive the required current and to squelch any back emf from the armature. DC supplies such as lead-acid supplies can deliver several thousand amperes for short duration, but are not practical for a large weapon since a large number are needed to provide the requisite voltage and current. Capacitors and compulsators can store very large amounts of energy and are capable of delivering hundreds of kilo-amperes. Capacitors store energy via an electric field; compulsators store energy mechanically in a flywheel. Compulsator stands for Compensated Pulsed Alternator; a compulsator uses a very low inductance generator to allow for rapid current rise and a high energy density flywheel to store energy. A compulsator can store enough energy to fire a ELS several consecutive times where a capacitor bank usually uses all of its energy on each shot and needs to be recharged after each shot. Compulsators generally can store more energy per unit volume than capacitors.

The development of switching circuit for very high current with a very low switching time of the order of micro-seconds is another problem in the ELS realization.
1.4.2 Rail Repulsion and Mechanical Wear

For any ELS the currents required will place large amounts of mechanical stress on the current carrying parts. The current carrying bars of the rail and the connectors must be stiff and fastened into place. If the ELS uses a plasma armature, the armature/projectile has to be tightly fit into the barrel and the barrel will have to be sealed to keep the plasma from escaping. For a solid armature the surface area in contact with the rails need to be maximized and good contact should be maintained. This is necessary to reduce arcing and spot welding and to allow for high current flow. But at the same time friction at the rail - armature interface also causes major problems in the ELS function. The deformation caused due to the electromagnetic forces developed can add to the problem by increasing the friction between surfaces in contact [12]. The technology of sliding electrical contacts for the conventional electric motors is well established. But here the speed is limited to a maximum of 100m/sec, whereas the ELS armatures move at very high speeds of the order of 1000 to 3000 m/sec or even more. It is difficult to design such sliding contacts, which can maintain good electrical and mechanical contacts at these speeds also [1]. Considerable research has gone into understanding the sliding contact characteristics at high velocities and also there has been proposals for different designs [13]-[15]. The armature rail interface should be designed to minimize gouging, and if any gouging is to occur, it is more desirable to have the damage on the armature instead of the rails, so that the rails can be reused for more launches. To achieve this, the rails should be as hard as feasible and the armature as soft as possible. Many materials and construction techniques have been tried to make long lasting rails. This is still an area of significant research. When the ELS is fired the armature/projectile should be injected at a high velocity to overcome that static friction of the barrel and to prevent spot welding. A fast injection also will spread out the heat generated across a greater area again helping to prolong rail life. Unless the magnetic field is supplied externally the armature should have a sufficient length of current carrying rail behind, before it contacts the rails, in order to allow a strong magnetic field to be created. Even then, it may be desirable to augment the magnetic field where the armature first makes contact since the current will
not immediately begin to accelerate the armature.

Various experiments have been conducted to study the mechanical friction, the viscous arc drag and the ablation effects. At the beginning of 1987 the French-German research institute of Saint-Louis (ISL) started its activities on electromagnetic acceleration. ISL has conducted research works on different kinds of fuse armatures for electromagnetic rail launchers [16]. In articles [17, 18], the influence of an armature material on acceleration dynamics has been investigated by means of computational methods. ELS housing also has got considerable importance in improving the efficiency and performance. Lehmann et. al [19] explains a comparative study on performance with different housing made of fiber wound composites.

1.4.3 Solid Armature Transition

Even though the early ELS research started with the use of metallic armatures, this technique was successful only for velocities up to about 1 km/s, as intermittent contact only could be maintained at hypervelocities. This caused the interface voltage between the armature and rails to rise from the few volts, typical of a metallic contact, to voltages of tens to hundreds of volts as arcing contacts caused plasmas to become the path for the current flow. This phenomenon is called transition [20]. Within the arc, temperatures up to 30,000 K have been postulated and serious erosion of rail surfaces has been observed. Pressures generated in the confined plasma region between the armature and the rails has been identified as a source of observed deformation of projectile components [21]. Maintaining non arcing contact to hypervelocities with metal armatures is important for improving efficiency, rail life and launch package integrity. Significant reduction in armature energy loss, which is identified as a major means of energy loss in ELS, can be achieved through the use of metal/transitioning armatures [22].

Several mechanisms that lead to armature transition have been identified and reported in the past and a review of possible armature transition mechanisms is given by J P Barber et. al [23]. This review indicates the complexity and multiplicity of mechanisms that can trigger the onset of transition. For ELS, Joule heating is another major concern. Since
roughly 50% of the breech electrical energy remains in the railgun after shot exit, rail heating is a significant concern [21].

1.5 Objectives

Though EML has so many advantages compared to conventional chemical rocket launch, the associated complexity is also more. There are numerous theoretical as well as experimental studies being conducted at different places across the world, to increase the launch velocity and to improve the efficiency of the system. In India, an initiative has been taken up at IISc towards the development of electromagnetic launch technology and a set of preliminary tests are being conducted. It is aimed to realize a table top ELS which can accelerate a projectile of around 50 g mass to 10m/s or more. Preliminary design has been completed and a few trials have already been carried out. This work is aimed at developing a three dimensional model of the system to understand various system parameters.

Lot of research work has gone into the modeling and simulation of ELS and special finite element codes are also developed for better representation of the coupled phenomena in ELS. The current density distribution through ELS rails and armature are extensively studied and many literature are available [24] - [28]. Elastic waves through the rails and change in contact pressure are studied in [29]. Experimental validation is also referred in many literatures. EMAS, MEGA and EMAP3D are the codes widely used [30],[31] and are copy righted codes available only to a core group. As none of these codes are available as such for public use, one has to develop own code or use some of the available multiphysics packages to do the modeling and simulation of ELS. More over most of the literatures rely on 2D approximation of the problem to predict the forces on armature and velocity induced [32]. So the present work was planned with the following objectives,

- Develop a 3D model of ELS using one of the commercially available software package COMSOL™.

- Study the design adequacy of the proposed ELS.
• Validation of the model using experimental results.

• Formulation of a 2D analytical model of system dynamics using Lagrangian formulation, incorporating the frictional force due to the moving armature.

• To study the characteristics of axial and flexural wave propagation through the rails.

• Spectral finite element formulation and analysis of system stability at various frequencies.

1.6 Overview

The report is organized as the following chapters. Chapter 2 reviews the current literature and puts in the current status of theoretical and experimental research work being conducted in this area. Modeling Analysis and design aspects are included as third chapter. Fourth chapter explains the 2D formulation for the system and stability analysis. The details of the experimental setup and trial runs are explained in chapter 5. A summary of the work is added towards the end with a section on the scope for future work.
Chapter 2

Review of Literature

2.1 Electromagnetic Launch

The scientific community once believed that ELS will not be successful in achieving hypervelocity. The prejudice was proved wrong by the experiments of Marshal et. al [33] in 1970s. In their trials, around 6km/s muzzle velocity was achieved for small projectiles. This triggered further research in the area of EML and hypervelocity and led to significant developments in the area. Many technical challenges previously regarded as bottle necks were solved by a series of theoretical studies, analysis, computations and experiments. ELS basically is a dynamic electro-mechanical system whose electromagnetic, structural and thermal properties are highly interdependent. To account for the behavior of transient electric and magnetic fields, fully time dependant three dimensional computational simulations are required. Several such efforts have come up from different universities and research institutes across the world, which have been successfully used for simulation of different systems. EMAP3D is one such code for simulation of thermo-mechanically coupled electromagnetic diffusion process,[34].
2.2 Recoil Effect

There is a debate between the old Ampere-Neumann electrodynamics and the modern relativistic electromagnetism [35]-[40] on the effect of recoil forces in ELS. As per the modern theory, vacuum can sustain large reaction forces and recoil forces will not be acting on the rails as such. The theory explains that the force experienced on the armature is literally exerted by local magnetic field pressure. The electromagnetic energy travels between the rails from the power source to the armature and the cause of the Lorentz force on the armature is the transference of the field energy momentum to the electrons in the metal. So the recoil force must cause a deceleration to the incoming field energy. Hence the recoil forces will not be felt on the rails [38, 39]. On the other hand Ampere-Neumann theory says that the recoil force will be acting in the longitudinal direction on rails and it can cause lateral deflection and buckling. This was proved through different experiments also. The Center for Electromagnetics Research of the North Eastern University, Boston has conducted experiments exclusively to prove the longitudinal recoil effect [40].

2.3 Material Development

Most of the components of an electromagnetic launch system can be improved with the use of advanced materials. A study by Urukov et. al [17] details the influence of material properties on the armature acceleration. Prasad et. al developed improved electrical conductors based on copper-silver alloys [41]. Because of their high strength-to-weight ratios, composites are a critical enabling material for high-energy storage and high-power pulsed alternators. Validation of the properties of composites is more complicated than for conventional metals. Test methods have been identified for the composite laminate materials, which are used in high-performance pulsed alternators [42]. Solid armatures are typically made of copper, or more commonly, aluminum, since they must have high electrical conductivity, be lightweight, but also be capable of high mechanical load-bearing capacity. Zielinski has evaluated the thermo-physical properties of an aluminum alloy, G1GAS 30, which is promising because of its increased mechanical properties [43].
comparative study of railgun housings done by Lehmann et. al [19] details the advantages of different materials for ELS housing like ceramics and composites. Katulka et. al has done a detailed analysis on the feasibility of using high temperature materials for ELS to improve the efficiency of the system [44]. S. Rosenwasser et. al has done commendable work on the insulator material selection for the ELS housing, which can improve the overall efficiency of the system [45].

2.4 Modeling and Analysis Efforts

Although substantial resources have been devoted to research in the area of ELS, practical implementation of such systems still faces some nontrivial technology challenges. One such challenge is the modeling of ELS with extreme and complex electromechanical environmental conditions, such as electromagnetic loading and ohmic heat source, which are three-dimensional, transient, nonuniform, and coupled to each other. A comparison of different formulations used for modeling ELS system is given by Rodger et. al [46].

Currently, only a limited number of computer codes are available, which are capable of simulating the ELS environment completely. Furthermore, each nation has access to only certain codes and each code has its own particular architecture and strengths. France and Germany ISL uses EMAS, the United Kingdom’s DRA choose MEGA, and the United States’ ARDEC uses EMAP3D. Recently there was an effort to coordinate the related research activities with some common standards and geometries. This has probably helped a lot towards the development and improvement of research tools for use in modeling and analysis of ELS [30, 34]. Another concern is the huge computing power required to solve such problems. The demand for more detailed understanding of the transient electromechanical processes has increased the requirement for fine mesh sizes in computational codes. The number of unknowns can easily reach a million. The solution of such problems may be beyond the capacity of even large single processor machines, requiring the use of a parallel approach such as massive parallel processing (MPP) [20].
2.5 Experimental Studies

Although there does not seem to be much in the way of documentation, it appears that the first electric gun was created in 1844. Nothing more is known of the device, or even how it worked. Clearly it did not fulfill the potential hoped for and did not replace conventional guns.[47]. In 1944, using batteries as his power source, Joachim Hansler created the first working railgun, which was able to propel a 10 g mass to speeds of about 1km/s. After this there was not much development and ELS remained more of a science fantasy than a reality until 1964 when the age of ELS truly began. That year, MB Associates used a 28kJ capacitor to accelerate nylon cubes with a plasma arc as the armature[48]. In this process, a fuse placed behind the projectile (nylon cube) was used to initially complete the circuit. As the current ran through the fuse it vaporized the metal and established the initial plasma arc. The plasma arc is moved forward by Lorentz’s Force, and pushes the nylon cube forward along with it. Later, in 1972 researchers at the Australian National University were able to accelerate 3 g projectiles to speeds of around 6km/s using plasma arcs and a 900kJ homopolar generator. Now developed countries like USA, UK etc. are conducting lot of research activities in this field and as part of it several experimental set ups are also established. Since then, there were many efforts by individuals and organizations in developing ELS. Along with that, a number of experiments were conducted in ELS and most of them were directed towards understanding the complex phenomena associated with EML, like armature transition, hypervelocity gouging, material wear, recoil effect on ELS etc.

Stefani and Parker [49] developed a hypothesis that in solid armature railguns, the onset of gouging is determined by the hardness of the harder material and by density and speed of sound of both the materials. Understanding of the mechanisms and control of wear is vital for successful development of solid armatures. The team has performed a set of experiments in which they measured the wear even at hypervelocities [50]. Additionally they denitrified a set of governing parameters that affect material loss and wear from armature. Another important phenomena studied in detail is the armature transition. Barber et. al [14] studied different mechanisms that can trigger transition and significant
rise in the transition onset velocity was achieved as a result of understanding the associated mechanisms. The improved understanding in electromagnetic launch is partially a consequence of significantly improved 3-D computational capability and partially due to clever and careful experiments with improved diagnostics. Stefani et al. measured melt-wave erosion on C-armatures launched in the medium caliber launcher (MCL) at IAT and recovered the armatures after flight [51]. Using a crowbar switch at the breech to rapidly shunt current out of the armature, they were able to interrupt the erosion and collect the armatures for analysis. In this way, they investigated the effects of current density and rail materials on the melt-wave erosion process. The debate on the action of recoil forces in ELS is still alive, though the experiments conducted by the Center for Electromagnetics Research of the North Eastern University, Boston has proved the action of recoil forces on the longitudinal rails. [40].
Chapter 3

Modeling, Analysis and Design

A miniature ELS is being developed at Indian Institute of Science (IISc), as an initial step towards the beginning of experimentation. The fig. 3.2 below shows the photograph of the system realized for experimentation.

3.1 ELS Design Aspects

The major design specification is the velocity of the projectile and the launch mass. The acceleration level has to be arrived at to impart the required velocity to the projectile and from this, the required force can be estimated. The following section explains the factors to consider for the design and the preliminary design calculations.

Step 1: Length of rails and launch time Assuming that the projectile is initially at rest, the acceleration that need to be imparted to it to reach the required muzzle velocity at the time of exit from the barrel can be estimated using basic mechanics and is given by $a = V^2/2S$, where V is the velocity of projectile and S is the length of the barrel. So the limiting factors here are the length of the rail and the acceleration level. The acceleration level depends on the time for which the force is applied to obtain the velocity; lesser the time higher the acceleration levels.

Fig. 3.1 shows the variation of acceleration levels and time of application of the force to obtain desired muzzle velocity, using a barrel of 1m length. For 6 km/s muzzle velocity,
Figure 3.1: Acceleration levels and time duration for which the force need to be applied to obtain specific muzzle velocity

The acceleration level is around \(1.75 \times 10^7 \text{m/s}^2\) and the time of application of the average force is less than 1ms. This time signifies the time for which the projectile remains in the barrel as the force will be applied only when the circuit through the rails is complete. In some cases like micro satellite launch, the acceleration level has to be limited to a particular level based on the payload requirements. In that case, the length of the rails needs to be larger to reduce the acceleration levels to the required level. Another issue here is the characteristic of the power supply. The applied potential has to reach the maximum value just before ejection and the power supply has to be designed to achieve this. So there should be a balance between the power supply and the length of the rails to extract maximum performance. Step 2: Calculation of force. Once the acceleration levels are fixed, the force required to accelerate the projectile in an ELS can be calculated using the Newton’s law \(F=ma\). This is the effective force acting on the projectile and hence it should be taken as the fraction of electromagnetic force acting on the projectile, considering the losses. Assuming a 25% loss, the actual force to be generated is 1.33 times
this force. The power supply has to be designed to deliver this average force for the time duration arrived at from the previous calculation.

## 3.2 ELS Assembly

![Figure 3.2: ELS system assembly (Power supply not shown)](image)

The scaled version of the assembly is made of copper rails and teak wood as insulation and the assembly is contained using SS clamps. Two copper rails of 60×40mm cross section and 1200mm length are used and the assembly has been realized with a bore of 40×40mm. The gun is designed to impart a velocity of around 100 m/s to a 'C' shaped aluminum armature of 50g weight. The required energy is supplied using a capacitor bank, charged via an external circuit. The power supply consisted of ten capacitors with a capacitance of 4700µF and peak voltage of 500V DC connected in parallel in a bank with their like terminals shorted by copper strips. The capacitor banks are connected to the rail through a copper bus bar assembly with a switching circuit and are charged using a transformer with an integrated rectifier assembly.
3.3 Governing Equations

The ELS works on the interaction between electric and magnetic fields and is governed by the two basic laws in electromagnetism, i.e the Biot-Savart’s Law and the Lorentz Force. Biot-Savart’s law gives the magnetic field produced by flow of current through a conductor and the equation is as follows:

\[ dB = \frac{\mu_0 I dl \times \hat{r}}{r^3}; \]  

(3.1)

where \( dB \) is the differential contribution to the magnetic field resulting from this differential element of wire, \( \mu_0 \) is the permeability of free space, \( I \) is the current, \( dl \) is a vector, whose magnitude is the length of the differential element of the wire, and whose direction is the direction of conventional current, \( \hat{r} \) is the displacement unit vector in the direction pointing from the wire element towards the point at which the field is being computed, and \( r \) is the distance from the wire element to the point at which the field is being computed.

The force on the current carrying conductor placed in a magnetic field is given by the Lorentz force expression. Lorentz force expression is given by

\[ F_E = q(E + V \times B); \]  

(3.2)

where \( F_E \) is the force, \( q \) is the charge, \( E \) is the electric field, \( V \) is the velocity of the charged particle, \( B \) is the magnetic field and \( \times \) represents vector cross product.

The ELS can be modeled as a system obeying the Maxwell’s relations, which describe the interaction between electric and magnetic fields. The equations converted to potential form are solved for the domain to get the scalar (\( \phi \)) and vector potentials (\( A \)). Time-dependant analysis of the system gives the values of \( A \) and \( \phi \), which can be used to calculate the current density distribution and to predict the body forces developed. This in turn can be used to check the muzzle velocity of the projectile. The model can be used to study the variation in exit velocity and the system efficiency with respect to the parameters like length of the rail, geometry of armature, rail cross section etc.
Maxwell's equations [52] are given by

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \]  
(3.3)

\[ \nabla \times \mathbf{H} = \mathbf{J} + \sigma \mathbf{E} + \epsilon \frac{\partial \mathbf{E}}{\partial t}, \]  
(3.4)

\[ \nabla \cdot \epsilon \mathbf{E} = \rho, \]  
(3.5)

\[ \nabla \cdot \mathbf{B} = 0, \]  
(3.6)

where, \( \mathbf{E} \) is the intensity of electric field, \( \mathbf{H} \) is the intensity of magnetic field, \( \mathbf{J} \) is the impressed current density, \( \epsilon \) is the permittivity, \( \mu \) is the permeability, \( \mathbf{B} \) is the magnetic flux density, \( \sigma \) is the conductivity of the medium.

The wave equations can be derived from these first order vector differential equations, which will give the spatial and temporal dependence of fields and the wave nature of the time varying electromagnetic fields. Substituting Eq.(3.4) to curl of Eq.(3.3) and putting \( \mathbf{B} = \mu \mathbf{H} \), we get

\[ \nabla^2 \mathbf{E} - \mu \sigma \frac{\partial \mathbf{E}}{\partial t} - \mu \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{1}{\epsilon} \nabla \rho + \mu \frac{\partial \mathbf{J}}{\partial t} \]  
(3.7)

Taking curl of Eq.(3.4), we get

\[ \nabla^2 \mathbf{H} - \mu \sigma \frac{\partial \mathbf{H}}{\partial t} - \mu \epsilon \frac{\partial^2 \mathbf{H}}{\partial t^2} = -\nabla \times \mathbf{J} \]  
(3.8)

Eqs.(3.7) and (3.8) forms the non homogenous generalized wave equations in a simple medium and for source free region with \( \sigma \neq 0, \rho=0 \) and \( \mathbf{J}=0 \), the above equation reduces to homogenous generalized wave equations

\[ \nabla^2 \mathbf{E} - \mu \sigma \frac{\partial \mathbf{E}}{\partial t} - \mu \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \]  
(3.9)
\[ \nabla^2 \mathbf{H} - \mu \sigma \frac{\partial \mathbf{H}}{\partial t} - \mu \epsilon \frac{\partial^2 \mathbf{H}}{\partial t^2} = 0 \]  

(3.10)

To simplify the mathematical analysis, auxiliary potential functions can be introduced. We have \( \nabla \cdot \mathbf{B} = 0 \). As divergent of \( \mathbf{B} \) is zero, it can be expressed the curl of a vector function say \( \mathbf{A} \). That is \( \mathbf{B} = \nabla \times \mathbf{A} \), where \( \mathbf{A} \) is called the magnetic vector potential. Substituting to Eq.(3.3) and rearranging the terms, we get

\[ \nabla \times \left( \mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} \right) = 0 \]  

(3.11)

which means that \( (\mathbf{E} + \partial \mathbf{A}/\partial t) \) is an irrotational function, which can be expressed as the gradient of a scalar potential. Let \( (\mathbf{E} + \partial \mathbf{A}/\partial t) = -\nabla \phi \), where \( \phi \) is the scalar potential and we get the

\[ \mathbf{E} = -\left( \nabla \phi + \frac{\partial \mathbf{A}}{\partial t} \right) \]  

(3.12)

Substituting \( \mathbf{H} = \mathbf{B}/\mu \) in Eq.(3.4), we get

\[ \nabla \times \mathbf{B} = \mu \epsilon \frac{\partial \mathbf{E}}{\partial t} + \mu \sigma \mathbf{E}; (As \mathbf{J} = 0) \]  

(3.13)

Substituting the expression for \( \mathbf{E} \) from Eq.(3.12) and putting \( \mathbf{B} = \nabla \times \mathbf{A} \), we get

\[ \nabla^2 \mathbf{A} - \mu \epsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} - \mu \sigma \frac{\partial \mathbf{A}}{\partial t} = \nabla (\nabla \cdot \mathbf{A}) + \mu \epsilon \nabla \frac{\partial \phi}{\partial t} + \mu \sigma \nabla \phi \]  

(3.14)

Also we have \( \nabla \cdot \mathbf{E} = 0 \) when charge density \( \rho = 0 \) and we get

\[ \nabla^2 \phi + \frac{\partial}{\partial t} \nabla \cdot \mathbf{A} = 0 \]  

(3.15)

To define the vector function \( \mathbf{A} \) we need to define the divergence also. Taking the choice \( \nabla \cdot \mathbf{A} = 0 \), which is known as Coulomb gauge, the equation reduces to

\[ \nabla^2 \mathbf{A} - \mu \epsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} - \mu \sigma \frac{\partial \mathbf{A}}{\partial t} = \mu \epsilon \nabla \frac{\partial \phi}{\partial t} + \mu \sigma \nabla \phi \]  

(3.16)
Equations. (3.16) and (3.17) represents the potential form of Maxwell’s equation for source free region with $J = 0$.

### 3.3.1 Muzzle Velocity of Projectile

The speed of a projectile is determined by several factors; the applied force, the amount of time that force is applied, and friction. Effect of friction can only be determined through testing. It is reasonable to assume a friction force equal to 25% of driving force. The projectile, experiencing a net force as per Eq.(1.1), will accelerate in the direction of that force. From Newton’s law, we have

$$a = \frac{F_E}{m};$$

where $a$ is the acceleration, $F_E$ is the force on projectile, $m$ is the mass of projectile.

As the projectile moves, the magnetic flux through the circuit is increasing and thus induces a back emf (electro motive force) manifested as a decrease in voltage across the rails. The theoretical terminal velocity of the projectile is thus the point where the induced emf has the same magnitude as the power source voltage, completely canceling it out. Eq.(3.19) shows the equation for the magnetic flux.

$$\phi_m = BA,$$

where, $\phi_m$ is the magnetic flux, $B$ is the magnetic flux density, $A$ is the area.

Induced voltage $V(i)$ is related to $\phi_m$ and the velocity of the projectile as per Eq.(3.20).

$$V(i) = \frac{d\phi_m}{dt} = B \frac{dA}{dt} = BW \frac{dx}{dt},$$

where $V(i)$ is the induced voltage, $d\phi_m/dt$ is the time rate of change in magnetic flux, $dx/dt$ is the velocity of the projectile, $W$ is the width of the rail.
Since the projectile will continue to accelerate until the induced voltage is equal to the applied, Eq.(3.21) shows the terminal velocity $v_{\text{max}}$ of the projectile.

$$v_{\text{max}} = \frac{V}{(BW)}; \quad (3.21)$$

where, $v_{\text{max}}$ is the terminal velocity of projectile, $V$ is the power source voltage.

These calculations give an idea of the theoretical maximum velocity of an ELS projectile, but the actual muzzle velocity is dictated by the length of the rails. The length of the rails governs how long the projectile experiences the applied force and thus how long it gets to accelerate. Assuming a constant force and thus a constant acceleration, the time at which the projectile leaves the rails and muzzle velocity (assuming the projectile is initially at rest) can be found using Eq.(3.22) and (3.23).

$$t_f = \sqrt{\frac{2Sm}{FE}}; \quad (3.22)$$

$$v_{\text{muzzle}} = \sqrt{\frac{2SF_E}{m}}; \quad (3.23)$$

where, $v_{\text{muzzle}}$ is the muzzle velocity, $S$ is the length of rails, $F_E$ is the force applied, $m$ is the mass of projectile.

These calculations ignore friction and air drag, and other associated losses due to arcing, gouging, excessive heating and plasma formation etc. Another important factor to be considered is the matching between rail length and the discharge characteristics of the power supply. If the rail length is short so that the projectile leaves the rails before the supplied energy is fully delivered, the system efficiency will be too less. On the other hand if the rail length is too long such that the projectile remains in the barrel even after the applied potential is dropped, there will be deceleration of the projectile.

### 3.4 Finite Element Simulation

None of the codes being used in countries like USA, France or Germany are available to the public to do the simulation. It has been noted that the COMSOL\(^{TM}\) multi
physics package can be successfully used in simulating the system. The latest version of this software has a model explaining the simulation of 3D railgun, which uses Induction current and Multimedia DC modules together to simulate the moving field for an ELS. The model assumes a constant potential applied across the rails and armature is modeled as a moving conductivity path. But in actual practice the applied potential will be a time varying one. Hence, here the PDE module of COMSOL™ is utilized in modeling the system, which is an interactive program for solving coupled PDEs in one or more physical domains simultaneously. In this study, the potential form of the Maxwell’s equations (Eq. 3.16 and 3.17) are solved. The armature is modeled as a static one as the primary aim is to assess the forces developed in the armature and the mechanical stress produced as a result of rail repulsion. The system of equations in the PDE General form of the module are

\[
e_a \frac{\partial^2 U}{\partial t^2} + d_a \frac{\partial U}{\partial t} + \nabla \cdot \Gamma = F \quad \text{in } \Omega \quad (3.24)
\]

\[
-\mathbf{n} \cdot (\Gamma) = G + \left( \frac{\partial R}{\partial U} \right)^T \mu \quad \text{on } \partial \Omega \quad (3.25)
\]

\[
0 = R \quad \text{on } \partial \Omega \quad (3.26)
\]

where \( \Omega \) is the computational domain: the union of all sub domains, \( \partial \Omega \) is the domain boundary and \( \mathbf{n} \) is the outward unit normal vector on \( \partial \Omega \).

Eq.(3.24) forms the system PDE, satisfied in domain \( \Omega \) and Eqs.(3.25) and (3.26) are the Neumann and Dirichlet boundary conditions respectively, which must be satisfied on domain boundary \( \partial \Omega \). \( \Gamma, F, G \) and \( R \) are coefficients. They can be functions of the spatial co-ordinates, the solution \( U \) or the space derivatives of \( U \). \( T \) represents the transpose and \( \mu \) is the Lagrange multiplier. The analysis is done in static as well as time dependent mode. The 3D geometry is modeled in COMSOL™ with an air column around. The model after meshing is shown in fig. 3.3. The air column around is suppressed for visualizing the geometry in the figure.

The first attempt was to obtain the results of a static analysis in which a constant applied potential is set between the rails and the FE solution is processed to get the
current density distribution and forces developed.

### 3.4.1 Static Analysis

Taking the variable $\mathbf{U} = \{ A_1 A_2 A_3 \phi \}^T$, where $A_1, A_2, A_3$ forms the components of $\mathbf{A}$ in the X, Y and Z directions, and setting the co-efficients to appropriate values as mentioned below will give the Maxwell’s equations (3.16) and (3.17) modeled in COMSOL$^\text{TM}$ PDE module.

In static analysis, the temporal derivatives in Eq.(3.24) will vanish and hence $e_a$ and $d_a$ need not be defined. Setting $\Gamma = \begin{bmatrix} A_{1x} & A_{1y} & A_{1z} \\ A_{2x} & A_{2y} & A_{2z} \\ A_{3x} & A_{3y} & A_{3z} \\ \phi_x & \phi_y & \phi_z \end{bmatrix}$ and $F = \begin{bmatrix} \mu \sigma \phi_x \\ \mu \sigma \phi_y \\ \mu \sigma \phi_z \\ 0 \end{bmatrix}$, where subscripts $(x)$, $(y)$ and $(z)$ represent the partial derivatives with respect to $x$, $y$ and $z$. 

![Figure 3.3: ELS 3D Model](image)
respectively, will represent the set of equations 3.16 and 3.17 for the static case, in the format of Eq.(3.24).

**Boundary Conditions**

Appropriate boundary conditions are; surface of air column and the rail surface which is perpendicular to current density, is magnetic insulation. i.e. $\mathbf{n} \times \mathbf{A} = 0$; and no potential ($\phi = 0$) condition for all the surfaces except the surface at which the potential is applied. For that surface, $\phi$ is replaced by the constant potential to be applied.

In COMSOL\textsuperscript{TM}, setting Dirichlet condition with coefficients, $G = \left\{ 0 \right\}_{4 \times 1}$ and $R = \left\{ A_3 - A_2, \ A_1 - A_4, \ A_2 - A_1, \ -\phi + V \right\}^T$, will simulate the condition. For the input surface, $V$ is the appropriate voltage and is zero for all other surfaces. All the internal boundaries will have a continuity condition for the magnetic field and electric insulation, i.e. $\mathbf{H}_2 - \mathbf{H}_1 = 0$ and $\mathbf{n} \cdot \mathbf{J} = 0$. In potential form this translates to $\mathbf{n} \times (\nabla \times \mathbf{A}) = 0$ and $\mathbf{n} \cdot \nabla \phi = 0$. In the model, this can be simulated by setting Neumann boundary condition with

$$G = \left\{ (-A_{1x} - A_{2x} - A_{3x}), \ (-A_{1y} - A_{2y} - A_{3y}), \ (-A_{1z} - A_{2z} - A_{3z}), \ 0 \right\}^T$$

Since the length of the rail will not considerably affect the distribution of electric and magnetic fields, a model with a length of 0.25 m for the rail is used for preliminary studies. The position of the armature is kept at the middle of the barrel. The model in COMSOL\textsuperscript{TM} is meshed with mesh refinement for rail-armature interface. The mesh consisted of a total of 2630 lagrange quadratic elements. The analysis is performed for a constant voltage of 300 V applied across the rails.

### 3.4.2 Time-dependant Analysis

The co-efficients $\Gamma$, $F$, $G$ and $R$ cannot take time derivatives of $\mathbf{U}$ and hence the equations cannot be modeled fully without going for state-space approach for variable $\mathbf{U}$. To simulate all the boundary conditions, $\mathbf{U}$ is taken as $\left\{ A_1 \ A_2 \ A_3 \ A_{1\text{dot}} \ A_{2\text{dot}} \ A_{3\text{dot}} \ \phi \ \phi_{\text{dot}} \right\}^T$, where $A_1, A_2, A_3$ forms the components of $\mathbf{A}$ in the X, Y and Z directions and $A_{1\text{dot}} A_{2\text{dot}} A_{3\text{dot}}$ and $\phi_{\text{dot}}$, respectively are the derivatives of $A_1, A_2, A_3$ and $\phi$ w.r.t time. By converting the basic Maxwell’s equations to the co-efficient form, the co-efficients in Eqs.(3.24 - 3.26)
takes the following form.

\[
e_a = \begin{bmatrix} -\mu \epsilon & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\mu \epsilon & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mu \epsilon & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix},
\]

\[
d_a = \begin{bmatrix} -\mu \sigma & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\mu \sigma & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mu \sigma & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \end{bmatrix},
\]

\[
\Gamma = \begin{bmatrix} A_{1x} & A_{2x} & A_{3x} & 0 & 0 & 0 & \phi_x & 0 \\ A_{1y} & A_{2y} & A_{3y} & 0 & 0 & 0 & \phi_y & 0 \\ A_{1z} & A_{2z} & A_{3z} & 0 & 0 & 0 & \phi_z & 0 \\ \end{bmatrix}^T,
\]

\[
F = \begin{bmatrix} \mu \epsilon \phi_{dotx} + \mu \sigma \phi_x & \mu \epsilon \phi_{doty} + \mu \sigma \phi_y & \mu \epsilon \phi_{dotz} + \mu \sigma \phi_z & A_{1dot} & A_{2dot} & A_{3dot} & 0 & \phi_{dot} \end{bmatrix}^T
\]

**Boundary conditions**

Conditions same as the static analysis case is applied here also, considering the time derivatives in Maxwell relation. Magnetic insulation for the surface of air column and the rail surface which is perpendicular to current density, and the zero and applied potentials for the input and output surfaces transforms to the following coefficients in time dependent PDE general form of COMSOL™.

\[
G = \begin{bmatrix} 0 \end{bmatrix}_{8 \times 1}, R = \begin{bmatrix} R_1 \ R_2 \ R_3 \ R_4 \ R_5 \ R_6 \ R_7 \ R_8 \end{bmatrix}^T,
\]

where

\[
R_1 = A_3 - A_2, \quad R_2 = A_1 - A_3, \quad R_3 = A_2 - A_1, \quad R_4 = A_{3dot} - A_{2dot},
\]

\[
R_5 = A_{1dot} - A_{3dot}, \quad R_6 = A_{2dot} - A_{1dot}, \quad R_7 = -\phi + V(t), \quad R_8 = -\phi_{dot} + \dot{V}(t),
\]

For the far field surfaces and for the output surface, \(R_7\) and \(R_8\) are zeros. For the input surface, \(V(t)\) is the appropriate voltage waveform and \(\dot{V}(t)\) is the derivative of \(V(t)\) w.r.t. time. Here also, all the internal boundaries will have a continuity condition for the magnetic field and electric insulation, i.e. \(H_2 - H_1 = 0\) and \(n.J = 0\). In potential
form this translates to $n.(\nabla \times A) = 0$ and $n.(\nabla \phi + \partial A/\partial t) = 0$ and setting Neumann boundary condition with

$$G = \left\{ G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_8 \right\}^T,$$

where $G_1 = -A_{1x} - A_{2x} - A_{3x}$, $G_2 = -A_{1y} - A_{2y} - A_{3y}$, $G_3 = -A_{1z} - A_{2z} - A_{3z}$, $G_4 = G_5 = G_6 = 0$, $G_7 = A_{1t} + A_{2t} + A_{3t}$, $G_8 = 0$, simulates the conditions.

### 3.5 Results and Discussions

#### 3.5.1 Static Analysis

The static analysis is performed for a constant potential of 300V applied across the rails. The results are discussed with reference to the fig.3.4 and 3.5. The surface marked with 1 is the surface to which positive potential is applied, which is referred as input rail and negative potential is applied to the surface marked with 2 and is referred as output rail.

![Figure 3.4: Reference cross section through the armature.](image)
The variation of $J_z$ along Z direction in the input rail is shown in fig.3.6. A maximum value of $2.5 \times 10^{10} A/m^2$ is observed for the rails till a distance of 0.1 m and between 0.1 and 0.15 m it drops down to zero. The armature is located at this position and the direction of current gets diverted to X direction in armature at this location and hence the result is as expected. From the theoretical value of resistivity for copper, which is the rail material, average current density is calculated and is coming to be $1.4 \times 10^{10} A/m^2$, which is of the same order. The fig.3.7 shows the variation of current density for the output rail 2. The values match with sign difference indicating reversal of direction.

The current through the armature is in the negative X direction and the fig. 3.8 shows the variation of current density through armature cross section (refer fig. 3.4) along X direction. The value is maximum as the X value reaches the armature cross section and the maximum value observed is $1.3 \times 10^{11} A/m^2$, which is matching with the theoretical calculation corresponding to the cross sectional area of the armature. Figs. 3.9(a) and 3.9(b) shows the surface plot of current density through the YZ plane. The plots corresponds to section located 5mm from the reference cross section in fig.3.4 towards positive and negative X directions.
Figure 3.6: Current density through the input rail along the reference section in fig.3.5.

Figure 3.7: Current density through the output rail along the reference cross section in fig.3.5.
The current density through the armature is much higher compared to that through the rails. This is expected as there is a reduction in cross section at the armature side. Fig.3.10 shows the streamline plot of $\mathbf{J}$ in the YZ plane. A slice plot of Z component of $\mathbf{J}$ is attached as 3.11. In all these plots, the concentration at the armature cross sections is clearly visible.

Forces developed on rail and armature are calculated by integrating the corresponding components of expression for Lorentz force (Eq.1.1) over the volume. It is calculated that a force of $2.7 \times 10^4 \text{kN}$ is developed on the armature. For the rail length of 1.2 m, and a projectile mass of 50g, the theoretical maximum velocity for no loss is calculated as per Eq.(3.23). The value comes out to about 36 km/s, which is on the higher side and can not be attained because of the following reasons:

- A constant potential is assumed across the rails through out the time, which is not possible in real case
- The rail length which forms the limiting factor for the duration of application of
Figure 3.9: Surface plots of current density through the armature. (a). Section 5mm towards the negative X direction from the reference cross section. (b). Section 5mm towards the positive X direction from the reference cross section.
The force estimated is on the higher side, as losses are neglected.

Figure 3.10: Current density through the rails and armature - Streamline plot in YZ plane.

Figure 3.11: Current density through the rails and armature - Slice plot.
3.5.2 Time-dependent Analysis

In the time dependent case, the analysis is performed for a duration of 3 ms with a time stepping of 0.5 ms. The actual discharge characteristics of the capacitor used for the experiments were not available and hence the simulation is tried out with a linear and exponential variation of applied potentials assumed across the rails, w.r.t time. The time duration is selected as per the discharge characteristics of the power supply as seen in literatures. In both the cases, the variation is formulated in such a way that the maximum applied potential is about 300V. Fig. 3.12 and 3.13 shows the variation of applied potential for both the cases. The results are comparable for both the cases as the applied potential variation is almost similar for both. Here the results are presented for the exponentially varying potential applied across the rails, which is the more realistic case.

![Electric Potential vs Time](image)

Figure 3.12: Applied potential across the rails - Linearly varied input.

The variation of current density through the rails along Z direction for different time steps are shown in Fig.3.14 and 3.15. As observed in the static case, the value drops to zero as the Z value reaches 0.13 which is the armature location. The maximum value is around $2.6 \times 10^{10} A/m^2$ which is matching with the maximum value corresponding to
constant potential applied across the rails. It is seen that there is a drop in the value along
Z, which shows the dissipation and at a particular Z location, the magnitude increases
with time, which is the reflection of increase in potential with time.

Similarly the variation of the current density through the armature cross section along
X direction is plotted for different time steps in Fig.3.16. For the half near to the input
side 1 (Ref. fig.3.4), i.e for the negative X co-ordinate side, the magnitude of $J_x$ decreases
with time and for the other half it increases with time. This is the clear indication of the
propagation with time.

For the exponentially varying potential also, forces coming on armature and rail are
calculated by integrating the Lorentz Force expression (Eq.1.1) over the volume and the
diagram plotting the variation of force on rails and armature in Z and X directions are shown
in fig.3.17 and 3.18.
Figure 3.14: Variation of $J_z$ through the input rail along Z direction for different time steps

Figure 3.15: Variation of $J_z$ through the output rail along Z direction for different time steps
Figure 3.16: Variation of $J_x$ through the armature along X direction for different time steps

Figure 3.17: Force on armature for the exponentially varying potential
By integrating the force-time curve and taking the average force acting on the arm, we can find out the velocity that will be imparted to a 50 g projectile. From the calculations, it is found that an average force of $796 \times 10^4$ N is acting on the armature. But this is true only if the armature leaves the ELS housing at 3 ms, as the forces will seize to act once the armature leaves the rail. Assuming the force is acting on the armature through out the discharge time, the velocity is calculated as per Eq.(3.23) and is found to be 19.5 km/s. However this is not practically feasible as the time duration for which the armature moving with a velocity of 19.5 km/s can remain in the barrel only for 0.4 ms, as per Eq.(3.22). This shows that the length of the rail need to be increased to extract the maximum available energy to propel the projectile. As the length is increased, the duration for which the force acting on the rails will be more and there should be a balance between the discharge characteristics of the power supply and the rail length so that the projectile leaves as soon as the applied potential reaches the maximum value. In this calculation of velocity, the friction, heating and plasma formation, and other associated loss are not considered.
and it is assumed that the entire force will be available to accelerate the projectile. The quantification of associated losses can be assessed only through experiments and with that data and the simulation results, a better approximation of the velocity can be arrived at.
Chapter 4

Analysis of System Stability

There are several problems associated with the dynamic instability of rails, coupled with the plasma formation due to friction and arcing. As discussed in sec.1.4.3, armature transition is one of the major problems observed in electromagnetic launchers, which can cause severe damage to the armature and the rails of the launcher. Several studies have been conducted on the possible explanations and studies by Johnson A J and Moon F C [29, 54], which explains the contribution of elastic waves through the guide rail in armature transition. Hence a similar study is envisaged here to explore the effect of frictional force between the armature and rails in the elastic wave propagation and its effect on system stability. The 2D analytical model is formulated by assuming the rails as a beam on elastic foundation, and a lagrangian approach is used for incorporating the friction due to relative movement of armature and rails.

4.1 Formulation

Here the conducting guide rails are modeled as an Euler Bernoulli beam and the rest of the support structure and mechanical clamps as a classic elastic foundation as shown in Fig. 4.1. $K_z$ represents the stiffness per unit length of the elastic foundation, and the assumed kinematics is that of Euler Bernoulli beam theory in which axial and transverse displacement fields are assumed as $u(x, y, z, t) = u_0(x, t) - z\partial v(x, t)/\partial x$ and $v(x, y, z, t) =$
$v(x, t)$, where $u$ and $v$ are the mid-plane axial and transverse displacements in the reference plane respectively.

![Simplified analytical model of ELS rail geometry](image)

Figure 4.1: Simplified analytical model of ELS rail geometry

The armature is moving in the X direction with a velocity $V_a$. The frictional force at the rail armature interface, $F_f$ is given by $\mu \times F_{zarm}$; where $F_{zarm}$ is the force on the armature in Z direction due to the electromagnetic interaction and $\mu$ is the coefficient of friction. The velocity of armature $V_a$ is given by Eq.(4.1).

$$V_a = \int \frac{F_{xarm}}{m_a} dt,$$  \hspace{1cm} (4.1)

where $F_{xarm}$ is the force on the armature in X direction, $m_a$ is the armature mass.

The rail can be considered as three segments as shown in Fig. 4.2, in which the middle segment is where the armature is positioned. The frictional force due to sliding contact will act as an offset load at the interface as shown in figure. Applying Hamilton’s first principle, one has

$$\delta \int_0^t (U - T + W) dt = 0$$  \hspace{1cm} (4.2)

where $U$ is the strain energy, $T$ is the kinetic energy and $W$ is the external work done;
\[ U = \int \frac{1}{2} \sigma^T C d\Omega, \quad T = \int \frac{1}{2} \rho (\dot{u}^2 + \dot{v}^2) d\Omega, \quad W = \sum_{i=0}^{L} f_x u_i + \sum_{i=0}^{L} f_z v_i + \sum_{i=0}^{L} M_x \]  

Figure 4.2: 2D Analytical model of rail

that is

\[ U = \int \frac{1}{2} \sigma^T C d\Omega, \quad T = \int \frac{1}{2} \rho \dot{u}^2 + \rho \dot{v}^2 d\Omega, \quad W = \sum_{i=0}^{L} f_x u_i + \sum_{i=0}^{L} f_z v_i + \sum_{i=0}^{L} M_x \]  

where \( \rho \) is the density of the rail material, \( f_x \) and \( f_z \) are the forces acting on the rails due to electromagnetic interaction, \( M_x \) is the moment due to the offset in frictional force given by \( F_f h/2 \), \( h \) being the depth of the rail. \( u_i \) and \( v_i \) are the corresponding displacements at the node \( 'i' \). To incorporate the relative movement of armature, lagrangian approach is used in the formulation, in which the time derivatives \( \partial/\partial t \) in Eq.(4.3) are replaced with total derivative \( D/Dt = \partial/\partial t - V_a \partial/\partial x \) [53].

The governing differential equations and the boundary conditions are obtained using the Hamilton’s principle in Eq. (4.2), which are expressed as

\[ \delta u : (EA - \rho AV_a^2) \frac{\partial^2 u}{\partial x^2} - \rho A \ddot{u} + 2\rho AV_a \frac{\partial \dot{u}}{\partial x} = 0; \]  

\[ \delta v : (EI + \rho IV_a^2) \frac{\partial^4 v}{\partial x^4} + \rho I \frac{\partial^2 \ddot{v}}{\partial x^2} - 2\rho IV_a \frac{\partial^3 \dot{v}}{\partial x^3} - 2\rho AV_a \frac{\partial \ddot{v}}{\partial x} + \rho AV_a^2 \frac{\partial^2 v}{\partial x^2} + \rho A \ddot{v} + K_z v = 0; \]

here the co-ordinate system is assumed through the midplane, which makes \( A_1 = \int \int zdydz \) zero and associated force and moment boundary conditions are

\[ \delta u : EA \frac{\partial u}{\partial x} - \rho AV_a (\ddot{u} - V_a \frac{\partial \dot{u}}{\partial x}) = F_f; \]
δv : \((EI + \rho IV_a^2) \frac{\partial^3 v}{\partial x^3} - 2\rho IV_a \frac{\partial^2 \dot{v}}{\partial x^2} + \rho AV_a \frac{\partial v}{\partial x} - \rho AV_a \ddot{v} + \rho I \frac{\partial \ddot{v}}{\partial x} = F_z; \quad (4.7)\)

\[ \delta(\partial v/\partial x) : (EI - \rho IV_a^2) \frac{\partial^2 v}{\partial x^2} + \rho IV_a \frac{\partial \dot{v}}{\partial x} = F_f h/2; \quad (4.8) \]

where, \([A I] = \int \int [1 z^2] dy dz\), \(E\) is the modulus of elasticity, \(\rho\) is the mass density of the rail material, \(K_z\) is the stiffness/unit length of the structural containment of rails, \(F_z\) is the Lorenz Force on rail in Z direction and \(F_f\) is the frictional force at the rail-armature interface.

Now that the governing equations for the assumed model are available, one can study the characteristics of the elastic wave propagation through the structure. This will help in identifying the effect of frictional force on wave propagation through the rails and also the possible instabilities that can be caused due to the friction. The Eqs.(4.4) and (4.5) are totally decoupled and hence are considered separately to study the behavior of axial and flexural wave propagation through the rails.

### 4.1.1 Flexural Wave Characteristics

Spectral formulation for the system can be arrived at by assuming a solution of the form \(v(x,t) = \sum \hat{v}_n(x,\omega)e^{i\omega_n t}\) and taking \(\hat{v}_n = B_n e^{-ik_n x}\) for Eq.(4.5), where \(k_n\) represents the wavenumber corresponding to \(n^{th}\) mode. Thus we get the characteristic equation for the flexural wave as a 4\(^{th}\) order polynomial in \(k_n\) given by Eq.(4.9).

\[ a_1 k_n^4 + a_2 k_n^3 + a_3 k_n^2 + a_4 k_n + a_5 = 0; \quad (4.9) \]

where,

\[ a_1 = EI + \rho IV_a^2, \quad a_2 = 2\rho IV_a \omega_n, \quad a_3 = \rho I \omega_n^2 - \rho AV_a^2, \quad a_4 = -2\rho AV_a \omega_n, \quad a_5 = K_z - \rho A \omega_n^2 \]

From Eq.(4.9), it is clear that there will be four distinct modes of waves for all non zero values of \(\omega_n\). As \(\omega_n \to 0\), the Eq.(4.9) will become a quadratic in \(k_n^2\), which will give two
conjugate pair of roots given by Eq.(4.10).

\[
k_n^2 = \frac{\rho A V_a^2}{2(EI + \rho IV_a^2)} \pm \frac{\sqrt{\rho^2 A^2 V_a^4 - 4K_z (EI + \rho IV_a^2)}}{2(EI + \rho IV_a^2)}
\]  \hspace{1cm} (4.10)

From Eq.(4.10), it is clear that for \( V_a = 0 \), the roots are purely imaginary and there is no wave propagation. For non-zero \( V_a \) the roots are complex and it shows the presence of all the four modes. As \( \omega_n \to \infty \), \( k_n \) is given by Eq.(4.11), which is the typical behavior of flexural waves in beams.

\[
k_n = \pm \sqrt{\frac{\rho A}{\rho I}}
\]  \hspace{1cm} (4.11)

The frequency at which there is no wave propagation is estimated by substituting \( k_n = 0 \) in Eq.(4.9) and is given by

\[
\omega_{cut-off} = \sqrt{\frac{K_z}{\rho A}}
\]  \hspace{1cm} (4.12)

The Eq.(4.9) can be solved for wavenumber \( k_n \) for a given material and geometry of the rails, over a range of frequency. The cut-off frequency for the given geometry is 10.07kHz. From Eq.(4.9), normal beam theory is recovered by neglecting friction and rotational inertia term and setting \( V_a \) to zero. The fig.4.3 shows the variation of wavenumber against frequency for this case. And for an Euler Bernoulli beam on elastic foundation, \( K_z \neq 0 \) and the rotational inertia term is neglected to get the typical behavior as shown in Fig.4.4 [55]. This shows that all the modes are complex below the cut off frequency and it is difficult to identify the forward moving wave.
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Figure 4.3: Variation of wavenumber with frequency for a normal Euler Bernoulli beam

Figure 4.4: Variation of wavenumber with frequency for an Euler Bernoulli beam on elastic foundation, neglecting rotational inertia
When rotational inertia is also considered in the characteristic equation, the plot of wavenumber changes as in Fig. 4.5. Here it can be seen that all the modes seize to propagate after a frequency value of around 60kHz. The effect of friction and armature velocity is studied by plugging in the value of $V_a$ calculated using the Eq.(4.1), as for a particular time the value of $F_{xarm}$ is known from the 3D finite element simulation. The value of $K_z$ is assumed as half the value of $E$ and the variation in wavenumber for each mode is plotted against frequency for different values of armature velocity $V_a$. Wavenumber corresponding to each mode is plotted for variation in $V_a$ and are shown in Figs. 4.6 to 4.9. It is seen that all the modes are complex except for the case with $V_a=0$ and all the modes propagate throughout the frequency range. Another fact to be noted is that the frequency at which the mechanical disturbances start propagating is decreasing as armature velocity increases.
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Figure 4.6: Variation of wavenumber $k_1$ with frequency for different values of $V_a$

Figure 4.7: Variation of wavenumber $k_2$ with frequency for different values of $V_a$
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Figure 4.8: Variation of wavenumber $k_3$ with frequency for different values of $V_a$

Figure 4.9: Variation of wavenumber $k_4$ with frequency for different values of $V_a$
4.1.2 Axial Wave Characteristics

Spectral formulation for longitudinal wave through the rails is arrived at by assuming a solution of the form \( u(x, t) = \sum \hat{u}_n(x, \omega) e^{i\omega_n t} \) and taking \( \hat{u}_n = A_n e^{-ik_n x} \) where \( k_n \) represents the wavenumber corresponding to \( n^{th} \) mode. The characteristic equation is a 2\textsuperscript{nd} order polynomial in \( k_n \) given by Eq.(4.21).

\[
a_1 k_n^2 + a_2 k_n + a_3 = 0; \tag{4.13}
\]

where,

\[
a_1 = \rho A - EA, \quad a_2 = i2\rho AV_n \omega_n, \quad a_3 = \rho A \omega_n^2
\]

and the corresponding wave numbers are \( k_{1,2} = \frac{\omega_n}{1-\frac{EA}{\rho A}} \left[ -iV_a \pm \sqrt{-V_a^2 - 1 + \frac{EA}{\rho A}} \right] \).

![Variation of wavenumber \( k_1 \) with frequency for different values of \( \mu \)](image)

It is evident that \( k_n \) becomes zero and longitudinal wave ceases to propagate for \( \omega_n = 0 \). When \( V_a = 0 \), \( k_{1,2} = \pm \omega_n \sqrt{EA/\rho A - 1} \), which means the wavenumbers are real and both modes will be propagating. The plot of wavenumber for each mode against frequency for
Figure 4.11: Variation of wavenumber $k_2$ with frequency for different values of $\mu$

different values of $V_a$ are shown in fig. 4.10 and 4.11. As $V_a$ increases, the wavenumbers become purely imaginary and there is no wave propagation.

From the figures it is clear that the real part of both the wavenumbers decreases as $V_a$ increases and becomes zero. In the plots for the case $V_a = 4000\,\text{m/s}$, the wavenumber is purely imaginary. This shows that the longitudinal wave propagates through the rails only for smaller values of $V_a$ and it is not predominant at higher armature velocities.

### 4.1.3 Dynamic Stiffness Matrix

The characteristics of elastic wave propagation through the guide rails is discussed in the previous sections. It is seen that the flexural mode is predominant at higher values of armature velocity. In order to study the effect of friction and armature velocity on the dynamic stability of the system, a spectral finite element model is formulated for the guide rail. A two element FE model is constituted, assuming fixed end conditions for the beam as shown in fig.4.12. The electromagnetic repulsive force is assumed to be acting at the
middle of the beam.

Assuming transverse displacement $\hat{v}(x, \omega)$ as

$$\hat{v}(x, \omega) = A e^{-ik_1x} + B e^{-ik_2x} + C e^{-ik_3x} + D e^{-ik_4x},$$  \hspace{1cm} (4.14)

Slope $\hat{\phi}(x, \omega)$ for Euler Bernoulli beam given by $d\hat{v}/dx$. Therefore,

$$\hat{\phi}(x, \omega) = R_1Ae^{-ik_1x} + R_2Be^{-ik_2x} + R_3Ce^{-ik_3x} + R_4De^{-ik_4x},$$  \hspace{1cm} (4.15)

where $R_1 = -ik_1$, $R_2 = -ik_2$, $R_3 = -ik_3$ and $R_4 = -ik_4$

Hence nodal displacements and slope for the first element, are

$$\begin{align*}
\begin{bmatrix}
\hat{v}_1 \\
\hat{\phi}_1 \\
\hat{v}_2 \\
\hat{\phi}_2
\end{bmatrix} &=
\begin{bmatrix}
e^{ik_1L/2} & e^{ik_2L/2} & e^{ik_3L/2} & e^{ik_4L/2} \\
R_1e^{ik_1L/2} & R_2e^{ik_2L/2} & R_3e^{ik_3L/2} & R_4e^{ik_4L/2} \\
1 & 1 & 1 & 1 \\
R_1 & R_2 & R_3 & R_4
\end{bmatrix}
\begin{bmatrix}
A_1 \\
B_1 \\
C_1 \\
D_1
\end{bmatrix}
\end{align*}$$  \hspace{1cm} (4.16)

where $\hat{v}_1$, $\hat{v}_2$, $\hat{\phi}_1$, $\hat{\phi}_2$ represents the nodal displacements and slope corresponding to nodes 1 and 2 and $A_1$, $B_1$, $C_1$, $D_1$ are the co-efficients corresponding to the first element.
This can be rearranged as

\[
\begin{bmatrix}
A_1 \\
B_1 \\
C_1 \\
D_1
\end{bmatrix} =
\begin{bmatrix}
\begin{array}{cccc}
e^{ik_1 L/2} & e^{ik_2 L/2} & e^{ik_3 L/2} & e^{ik_4 L/2} \\
R_1 e^{ik_1 L/2} & R_2 e^{ik_2 L/2} & R_3 e^{ik_3 L/2} & R_4 e^{ik_4 L/2} \\
1 & 1 & 1 & 1 \\
R_1 & R_2 & R_3 & R_4
\end{array}
\end{bmatrix}
^{-1}
\begin{bmatrix}
\hat{v}_1 \\
\hat{\phi}_1 \\
\hat{v}_2 \\
\hat{\phi}_2
\end{bmatrix}
\] (4.17)

Similarly for the second element

\[
\begin{bmatrix}
\hat{v}_2 \\
\hat{\phi}_2 \\
\hat{v}_3 \\
\hat{\phi}_3
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 & 1 \\
R_1 & R_2 & R_3 & R_4 \\
e^{-ik_1 L/2} & e^{-ik_2 L/2} & e^{-ik_3 L/2} & e^{-ik_4 L/2} \\
R_1 e^{-ik_1 L/2} & R_2 e^{-ik_2 L/2} & R_3 e^{-ik_3 L/2} & R_4 e^{-ik_4 L/2}
\end{bmatrix}
\begin{bmatrix}
A_2 \\
B_2 \\
C_2 \\
D_2
\end{bmatrix},
\] (4.18)

where \(\hat{v}_2, \hat{v}_3, \hat{\phi}_2, \hat{\phi}_3\) represents the nodal displacements and slope corresponding to node 2 and 3 and \(A_2, B_2, C_2, D_2\) are the coefficients corresponding to the second element.

Eq.(4.18) on rearranging gives

\[
\begin{bmatrix}
A_2 \\
B_2 \\
C_2 \\
D_2
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 & 1 \\
R_1 & R_2 & R_3 & R_4 \\
e^{-ik_1 L/2} & e^{-ik_2 L/2} & e^{-ik_3 L/2} & e^{-ik_4 L/2} \\
R_1 e^{-ik_1 L/2} & R_2 e^{-ik_2 L/2} & R_3 e^{-ik_3 L/2} & R_4 e^{-ik_4 L/2}
\end{bmatrix}
^{-1}
\begin{bmatrix}
\hat{v}_2 \\
\hat{\phi}_2 \\
\hat{v}_3 \\
\hat{\phi}_3
\end{bmatrix}
\] (4.19)

Substituting \(v(x, t) = \sum \hat{v}_n(x, \omega_n e^{i\omega_n t}\) in Eqs.(4.7) and (4.8) gives the expression for shear force \(\hat{V}(x, \omega)\) and bending moment \(\hat{M}(x, \omega)\) as follows

\[
\hat{V}(x, \omega_n) = (EI + \rho IV_a^2 \frac{d^3 \hat{v}}{dx^3}) - i2\rho IV_a \omega_n \frac{d^2 \hat{v}}{dx^2} + (\rho AV_a^2 - \rho I \omega_n^2) \frac{d\hat{v}}{dx} - i\rho AV_a \hat{\phi},
\]

\[
\hat{M}(x, \omega_n) = (EI - \rho IV_a^2 \frac{d^2 \hat{v}}{dx^2}) + i\rho IV_a \omega_n \frac{d\hat{v}}{dx},
\]

For element 1, the \(\hat{V}, \hat{M}\) at the node points are obtained as
where
\begin{align*} 
c_{11} &= -i [(EI + \rho IV_a^2)k_1^2 + 2\rho IV_\omega k_1^2] - (\rho AV_a^2 - \rho I\omega_n^2)k_1 - \rho AV_a \\
c_{12} &= -i [(EI + \rho IV_a^2)k_2^2 + 2\rho IV_\omega k_2^2] - (\rho AV_a^2 - \rho I\omega_n^2)k_2 - \rho AV_a \\
c_{13} &= -i [(EI + \rho IV_a^2)k_3^2 + 2\rho IV_\omega k_3^2] - (\rho AV_a^2 - \rho I\omega_n^2)k_3 - \rho AV_a \\
c_{14} &= -i [(EI + \rho IV_a^2)k_4^2 + 2\rho IV_\omega k_4^2] - (\rho AV_a^2 - \rho I\omega_n^2)k_4 - \rho AV_a \\
c_{21} &= (EI - \rho IV_a^2)k_1^2 - \rho IV_\omega k_1 \\
c_{22} &= (EI - \rho IV_a^2)k_2^2 - \rho IV_\omega k_2 \\
c_{23} &= (EI - \rho IV_a^2)k_3^2 - \rho IV_\omega k_3 \\
c_{24} &= (EI - \rho IV_a^2)k_4^2 - \rho IV_\omega k_4 \\
\end{align*}

Similarly for second element, we can write

\begin{align*} 
\begin{bmatrix} \hat{V}_1 \\ \hat{M}_1 \\ \hat{V}_2 \\ \hat{M}_2 \end{bmatrix} &= \begin{bmatrix} c_{11}e^{ik_1L/2} & c_{12}e^{ik_2L/2} & c_{13}e^{ik_3L/2} & c_{14}e^{ik_4L/2} \\ c_{21}e^{ik_1L/2} & c_{22}e^{ik_2L/2} & c_{23}e^{ik_3L/2} & c_{24}e^{ik_4L/2} \\ -c_{11} & -c_{12} & -c_{13} & -c_{14} \\ -c_{21} & -c_{22} & -c_{23} & -c_{24} \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{bmatrix}, 
\end{align*} \tag{4.22}

where the constants take the same value as in Eq.(4.22)

Substituting the expressions for co-efficients \(A_1, B_1, C_1, D_1\) from Eq.(4.17) to Eq.(4.22) and \(A_2, B_2, C_2, D_2\) from Eq.(4.19) to Eq.(4.23), \(\hat{V}_n\) and \(\hat{M}_n\) can be expressed in terms of the nodal displacements and slopes, which gives the individual stiffness matrix for the elements. And by assembling the individual stiffness matrix for each element, we get the global stiffness matrix of the form

\begin{align*} 
\begin{bmatrix} \hat{V}_1 \\ \hat{M}_1 \\ \hat{V}_2 \\ \hat{M}_2 \\ \hat{V}_3 \\ \hat{M}_3 \end{bmatrix}^T &= \begin{bmatrix} K \end{bmatrix}_{6 \times 6} \begin{bmatrix} \hat{v}_1 \\ \hat{\phi}_1 \\ \hat{v}_2 \\ \hat{\phi}_2 \\ \hat{v}_3 \\ \hat{\phi}_3 \end{bmatrix}^T \tag{4.24}
\end{align*}
Applying the fixed end boundary conditions, i.e. displacement and slope at nodes 1 and 3 \((\hat{v}_1, \hat{\phi}_1, \hat{v}_3, \hat{\phi}_3)\) as zeros, the corresponding rows and columns in the stiffness matrix is eliminated to get the reduced stiffness matrix of the form

\[
\begin{bmatrix}
\hat{V}_2 \\
\hat{M}_2
\end{bmatrix}^T = 
\begin{bmatrix}
K_R
\end{bmatrix}_{2\times2}
\begin{bmatrix}
\hat{v}_2 \\
\hat{\phi}_2
\end{bmatrix}^T
\tag{4.25}
\]

By solving the determinant of the reduced stiffness matrix, we can find out the natural frequencies of the system. The value of determinant of the reduced stiffness matrix is plotted on log scale against frequency for different values of \(K_z\) and is shown in Fig. 4.13, for \(V_a=0\). It is seen from the plots that the first mode natural frequency is increasing as \(K_z\) increases. This indicates that the instabilities are less as the stiffness of the structural containment increases. The fig.4.14 shows the variation in frequency with respect to \(K_z\), expressed as a percentage of \(E\). It can be noted that the change in frequency is more at lower \(K_z\) values and the curve becomes almost flat for values of \(K_z\) above 50% of \(E\).
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Figure 4.14: Change in first mode frequency with stiffness of structural containment

As the $K_z$ value reaches 50% of the modulus, the first mode frequency appears to be around 400kHz. As it is reasonable to assume this value for $K_z$, the effect of armature velocity is studied for this case. The variation in the first mode frequency for different values of $V_a$ is plotted in fig.4.15. It is seen that as the armature velocity increases, the first mode frequency value comes down, which means that the resulting instabilities will be more.

The fig.4.16 shows the variation in frequency with respect to $V_a$. It is observed that the drop in frequency is more at lower $V_a$ values and the curve shows an asymptotic behavior. At $V_a$ value of 6km/s, the corresponding frequency is 130kHz, which is almost the same value corresponding to $K_z$ equal to 5% of the modulus, when the armature velocity is zero. This indicates that the stiffness of the containment should be very high to avoid instabilities at higher armature velocities.
CHAPTER 4. ANALYSIS OF SYSTEM STABILITY

Figure 4.15: Determinant of reduced stiffness matrix against frequency for various values of armature velocity.

Figure 4.16: Variation in first mode frequency with armature velocity.
Chapter 5

Experimental Setup

The development activities for a table top ELS for launching small projectiles of 50g size is in progress at the high enthalpy laboratory of Department of Aerospace Engineering at IISc. The basic idea is to study various issues associated with the launch and to understand the complex physical phenomena associated with EML, like recoil, armature transition, effect of various system parameters on the performance and efficiency of the system etc. The following are the preliminary design specifications for the system under development.

- Length of rails < 2 m
- Projectile mass ≃ 50 g
- Muzzle Velocity > 10 m/s
- Muzzle Energy > 2.5 J
- Average Acceleration > 25 m/s²
- Launch Time < 0.4 ms

5.1 ELS Components

Preliminary design is completed with the following components for the system.
5.1.1 Power Supply

As explained in the previous sections, the ELS requires a quick discharge of a large current and several kA of current has to be supplied in a short time to accelerate the projectile. Various systems can be used for the power supply. Here a set of capacitors in parallel are used to supply the necessary power. The capacitor bank is charged via an external circuit and 10 nos of electrolytic capacitor with rated capacitance of $4700 \mu F$ and peak voltage of 500V DC were included. The power supply scheme and the charging circuit are shown in Fig.5.1. Manual switching was used for the power supply.

![Figure 5.1: Schematic of the power supply scheme for table top ELS](image)

5.1.2 Barrel and Rails

As for the initial tests a rail barrel system with a bore of $40 \times 40$mm and length 1.2m is realized. Copper is selected as the material for rails as it is the universal choice due to its high conductivity and adequate mechanical strength. Wood is opted as the insulator material for separating the rails due to availability and manufacturability. The realized
barrel is shown in fig. 3.2 and a schematic cross section is included in fig. 5.2. The tolerances
achieved in the bore dimensions play an important role in ELS, as clearance between rails
and armature will lead to arcing and affect the system performance. On the other hand,
an interference fit can cause the armature to get struck inside the bore also. Care is taken
to ensure the correctness and uniformity in bore size. But there were facility limitations
in the measuring and inspecting the assembled geometry.

![Schematic of the ELS barrel realized](image)

**Figure 5.2: Schematic of the ELS barrel realized**

### 5.1.3 Armature

The armature was milled out from aluminium 6061-T9 block through CNC machining to
obtain the required precision. It is important to control the geometry of the armature
also to allow sufficient contact between the armature and rails.

### 5.2 Test Runs

Three test runs were conducted using the hardware realized. In the first two tests, a
miniature circuit breaker was used to switch on the supply. High arching was observed
and the armature was not ejected out. Later it is observed that the armature got spot welded to the rails at the initial position.

In the third run, the switch was replaced with a 3 pole knife switch, which is again a manually operated switch with high current rating. Very high arcing was observed at the switch and the switch got damaged. This time also armature did not move from initial position and spot welds were observed at the armature location during post test inspection. The following are the possible causes of failure inferred from the tests

- A good share of the energy is wasted in arcing due to slow manual switching. The solution is to use a faster switching circuit with the required current rating. A faster switching circuitry is under development.

- The static friction between the rail and the armature might have blocked the armature from movement initially, which can lead to spot welding. The solution is to use an injector to push in the armature in synchronization with the power supply switching.

- The bore dimension were assembly dependant as per the design and needed precisely machined components to keep the armature in contact with the rails. With the realized hardware the tolerances achieved was not good and this also might have resulted in fine gap between the armature and rails, which in turn can initiate arcing and spot welding. An improvement over the system assembly is planned and the realization is in progress.

5.3 ELS with Composite Housing

The total assembly is shown in Fig.5.3 and following are the major components of the assembly.

**Enclosure:** The enclosure is in two piece, made of molded 10mm thick FRP and ribs are provided to improve the structural integrity. Dovel-pins are included to guide during assembly process.
**Spacers:** It forms the separating part between the rails, the dimensions is important in deciding the bore size. 5mm thick ribbed FRP with Polyurethane foam sandwiched core is used as spacer.

**Insulators:** Insulators fill the gap between the housing and the rails and ribbed FRP with 10mm/5mm walls are planned as spacers. Considering the requirement of measurement during future tests, threaded holes are provided for inserting sensors.
Chapter 6

Conclusions and Future Scopes

An attempt has been made to simulate the ELS by modeling in the PDE module of the COMSOL\textsuperscript{TM} multi physics software. The simulation of electromagnetic interaction is modeled using the potential form of Maxwell’s equations and the field distributions are simulated. The forces coming on the armature and rails are also evaluated, which in turn are used to predict the attainable velocity for the projectile. It is seen that the static analysis is not sufficient to predict the forces and velocity. In the time dependant study, the armature is assumed to be fixed at a particular location, which forms a quasi-steady approximation of the problem. The forces and velocity are calculated for a linearly varying potential and for an exponential variation. The results shows a conservative estimate of velocity up to 19km/s with an applied potential of 300V in 3ms for the exponentially varying potential. Thus the possibility of using COMSOL\textsuperscript{TM} for the simulation of electromagnetic phenomena is established and the same can be used for the design improvement, once the other modules are added to the model for structural and thermal analysis.

An analytical study on the system dynamics is performed assuming Euler Bernoulli theory with Lagrangian formulation to incorporate the frictional force due to the moving armature. Characteristics of flexural and axial wave propagation is also studied and found that the flexural wave propagation is predominant and all the four modes are present when armature velocity in non-zero. Spectral finite element formulation is done to derive the dynamic stiffness matrix as a function of frequency and the stability of the system is
assessed for different frequency regimes. The study showed that the instabilities are more for lower values of stiffness of structural containment and for higher armature velocities.

6.1 Scope for Future Work

This work was a preliminary estimate and the experimental validation of the results were also aimed at, which could not be completed for want of time. Few initial runs test runs tried out were not successful. So the next step shall be to validate the force and acceleration levels from test results. In the simulation, applied potential characteristics is assumed as an exponential discharge and the simulation can be repeated for appropriate power supply characteristics later, once the design is finalized. ELS basically is a dynamic electro-mechanical system whose electromagnetic, structural and thermal properties are highly interdependent. To account for the coupled behavior, the model can be extended with other modules for incorporating structural and thermal responses. The system is modeled as a quasi-static one, with the armature position fixed with respect to the rails and this is sufficient for a preliminary estimate. But for more detailed studies, the possibility of incorporating armature movement in the model need to be explored.

In the laboratory experiments it was tried to propel a stationary armature, which can probably be the reason to get it welded to the rail surface. Hence another module to inject the armature in synchronization with the power switching may be added to overcome the problem associated with static friction. Extensive tests can be planned to assess the system performance, effect of recoil etc. once the trial runs become successful.
Bibliography


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